Numeric Response Questions

Differentiation

Q.1 If the derivative of f(tan x) w.r.t. $g(\sec\ x)$ at $x=\pi/4$, where $f'(1)=2$ and $g'(\sqrt{2})=4$ is $\frac{\sqrt{2}}{k}$ then find k ?

Q.2 Lat $\phi(x)$ be the inverse of the function $f(x)$ and $f'(x)=\frac{1}{1+x^5}$ and $\frac{d}{dx}\phi(x)=[1+\phi(x)]$ then find n.

Q.3 Let y be a function of x, such that $\log(x+y)-2xy=0$, then find $y'(0)$.

Q.4 Let $f(x), g(x)$ be two continuously differentiable functions satisfying the relationships $f'(x)=g(x)$ and $f'(x)=-f(x)$. Let $h(x)=[f(x)]^2+[g(x)]^2$, If $h(0)=5$, then find value of $h(10)$

Q.5 If $5f(x)+3f(1/x)=x+2$, then find $\frac{d}{dx}(x, f(x))$ at $x=1$.

Q.6 If $y=\tan^{-1}\left(\frac{2^x}{1+2^{2x+1}}\right)$, then $\frac{dy}{dx}$ at $x=0$ is $k\log\frac{1}{2}$ then find k.

Q.7 Find the first derivative of the function $\left[\cos^{-1}\left(\sin\sqrt{\frac{1+x}{2}}\right)+x^x\right]$ with respect to x at $x=1$.

Q.8 Let $g(x)$ be inverse of function $f(x)=x^3+2x^2+4x+\sin\left(\frac{\pi}{2}x\right)$ and $g'(8)$ is equal to $\frac{1}{k}$ then find k.

Q.9 Find $\frac{d}{dx}\left[\sin^2\ \cot^{-1}\left\{\sqrt{\frac{1-x}{1+x}}\right\}\right]$

Q.10 If $y=\tan^{-1}\frac{\log(e/x^2)}{\log(ex^2)}+\tan^{-1}\frac{3+2\log x}{1-6\log x}$, then find $\frac{d^2y}{dx^2}$.

Q.11 If $f(x)=\cot^{-1}\left(\frac{x^x-x^{-x}}{2}\right)$, then find $f'(1)$.

Q.12 If $y=2^{ax}$ and $\frac{dy}{dx}=\log\ 256$ at $x=1$, then find value of a

Q.13 If $f(x)=\cos(x^2-2[x])$ for $0<x<1$ and $f'\left(\frac{\sqrt{\pi}}{2}\right)=-\sqrt{\frac{\pi}{k}}$, then find k.

Q.14 If $y=\log_{e^5}(x-3)^2$ and $x\neq 0$, then find $\left(\frac{dy}{dx}\right)_{x=4}$,

Q.15 If $y=(1+x)(1+x^2)(1+x^4)\ldots(1+x^{2^6})$ then find $\frac{dy}{dx}$ at $x=0$.

ANSWER KEY

1. 2.00	**2.** 5.00	**3.** 1.00	**4.** 5.00	**5.** 0.87	**6.** 0.60	**7.** 0.75
8. 11.00	**9.** 0.50	**10.** 0.00	**11.** –1.00	**12.** 2.00	**13.** 2.00	**14.** 0.50
15. 1.00						

Hints & Solutions

1. Let u = f (tan x) and v = g(sec x)

$\Rightarrow \frac{du}{dx} = f'(\tan x)\sec^2 x$

and $\frac{dv}{dx} = g'(\sec x)\sec x \tan x$

$\Rightarrow \frac{du}{dv} = \frac{du}{dx} \Big/ \frac{dv}{dx} = \frac{f'(\tan x)\sec^2 x}{g'(\sec x)\sec x \tan x}$

$\Rightarrow \left[\frac{du}{dv}\right]_{x=\frac{\pi}{4}} = \frac{f'\left(\tan\frac{\pi}{4}\right)}{g'\left(\sec\frac{\pi}{4}\right)\sin\frac{\pi}{4}} = \frac{f'(1)\sqrt{2}}{g'(\sqrt{2})}$

$= \frac{2\sqrt{2}}{4} = \frac{1}{\sqrt{2}}$

2. $\phi(x) = f^{-1}(x)$

$f(\phi(x)) = x$

$\Rightarrow f'(\phi(x))\,\phi'(x) = 1$

$\Rightarrow \phi'(x) = \frac{1}{f'(\phi(x))} = 1 + (\phi(x))^5$

3. $\log(x+y) - 2xy = 0$

put x = 0 we get y = 1

on differentiating $\log(x+y) - 2xy = 0$

we get

$y' = -\left[\frac{\frac{1}{x+y} - 2y}{\frac{1}{x+y} - 2x}\right]$

put x = 0, y = 1

$y'(0) = 1$

4. $h'(x) = 2f(x)f'(x) + 2g(x)g'(x)$

$= -2f(x)f''(x) + 2f(x)f''(x)$

$(\because g(x) = f'(x),\ g'(x) = f''(x))$

$h'(x) = 0$

h (x) = constant

$\Rightarrow \left.\begin{matrix} h(0)=5 \\ h(10)=5 \end{matrix}\right]$

5. $5f(x) + 3f(1/x) = x + 2$ .... (i)

put $x \rightleftharpoons \frac{1}{x}$

$5f(1/x) + 3f(x) = \frac{1}{x} + 2$ .... (ii)

(i) × 5 – (ii) × 3

7. $y = \frac{\pi}{2} - \sin^{-1}\sin\sqrt{\frac{1+x}{2}} + x^x$

$= \frac{\pi}{2} - \sqrt{\frac{1+x}{2}} + x^x$

$\frac{dy}{dx} = 0 - \frac{1\times(1/2)}{2\sqrt{\frac{1+x}{2}}} + x^x(1+\log x)$

at x = 1 $= -\frac{1}{4} + 1 = 3/4$

8. $\because f(1) = 8$

$\& f'(x) = 3x^2 + 4x + 4 + \frac{\pi}{2}\cos\frac{\pi x}{2}$

$\because$ Let $g(x) = f^{-1}(x)$

$f(g(x)) = x$

$f'(g(x)).\, g'(x) = 1$

$g'(8) = \frac{1}{f'(g(8))} = \frac{1}{f'(1)} = \frac{1}{11}$

9. put $x = \cos\theta$

10. $y = \tan^{-1}\frac{\log(e/x^2)}{\log(ex^2)} + \tan^{-1}\frac{3+2\log x}{1-6\log x}$

$= \tan^{-1}\frac{(\log e - \log x^2)}{(\log e + \log x^2)} + \tan^{-1}\frac{3+2\log x}{1-6\log x}$

$= \tan^{-1}\left(\frac{1-2\log x}{1+2\log x}\right) + \tan^{-1}3 + \tan^{-1}\log x^2$

$= \tan^{-1}(1) - \tan^{-1}\log x^2 + \tan^{-1}3 + \tan^{-1}\log x^2$

$= \tan^{-1}(1) + \tan^{-1}(3)$

$\frac{d^2y}{dx} = 0$

11. $f(x) = \cot^{-1}\left(\frac{x^x - x^{-x}}{2}\right)$

$f'(x) = \frac{-1}{1+\left(\frac{x^x - x^{-x}}{2}\right)^2} \times \left(\frac{x^x(1+\log x) + x^{-x}(1+\log x)}{2}\right)$

$f'(1) = \frac{-1}{1+0} \times \frac{1(1+0)+1(1+0)}{2} = -1$

12. $\log y = ax \log 2$

$\frac{1}{y}\frac{dy}{dx} = a \log 2$

$\frac{dy}{dx} = 2^{ax}.a \log 2$

$\left.\frac{dy}{dx}\right)_{x=1} = 2^a.\ a \log 2 = 8 \log 2$

$= 2^a.a = 8;\ a = 2$

13. $f(x) = \cos(x^2)$

$f'(x) = 2x \sin x^2$

$f'\left(\frac{\sqrt{\pi}}{2}\right) = -\sqrt{\frac{\pi}{2}}$

14. $y = \frac{\log_e(x-3)^2}{\log_e e^x} \Rightarrow y = \frac{2\log_e(x-3)}{x}$

$\frac{dy}{dx} = \frac{\frac{2}{(x-3)}.x - 2\log_e(x-3)}{x^2}$

$\left.\frac{dy}{dx}\right]_{x=4} = \frac{\frac{2\times 4}{4-3} - 2\log_e(4-3)}{16} = \frac{8}{16} = \frac{1}{2}$

15. $y = (1+x)(1+x^2)(1+x^4) \dots (1+x^{2^n})$

Multiplying numerator and denominator by $(1-x)$

$\Rightarrow y = \frac{(1-x)(1+x)(1+x^2)\dots\dots(1+x^{2n})}{(1-x)}$

$\Rightarrow y = \left(\frac{1-x^{2^{n+1}}}{(1-x)}\right)$

$\therefore \frac{dy}{dx}$

$= \frac{(1-x).\left\{-2^{n+1}.x^{2^{n+1}-1}\right\} - \left(1-x^{2^{n+1}}\right)(-1)}{(1-x)^2}$

So $\left.\frac{dy}{dx}\right|_{x=0} = \frac{-2^{n+1}.0.1+1-0}{1^2} = 1$